Suggested Solution to Project 5

Populations of Birds and Insects on Pöel Island

a) x – insects, y – birds. In the absence of birds (y=0) the population of insects will increase exponentially (equation 1). In the absence of insects (x=0) which are food for birds the population of birds will decrease exponentially (equation 2).

b) dx/dt = 0 and dy/dt=0 gives x = 0; y = 0 and x = 25,000; y = 200. The first equilibrium is trivial. The second equilibrium means that both populations are in balance: 25,000 insects are just enough to feed 200 birds.

c)

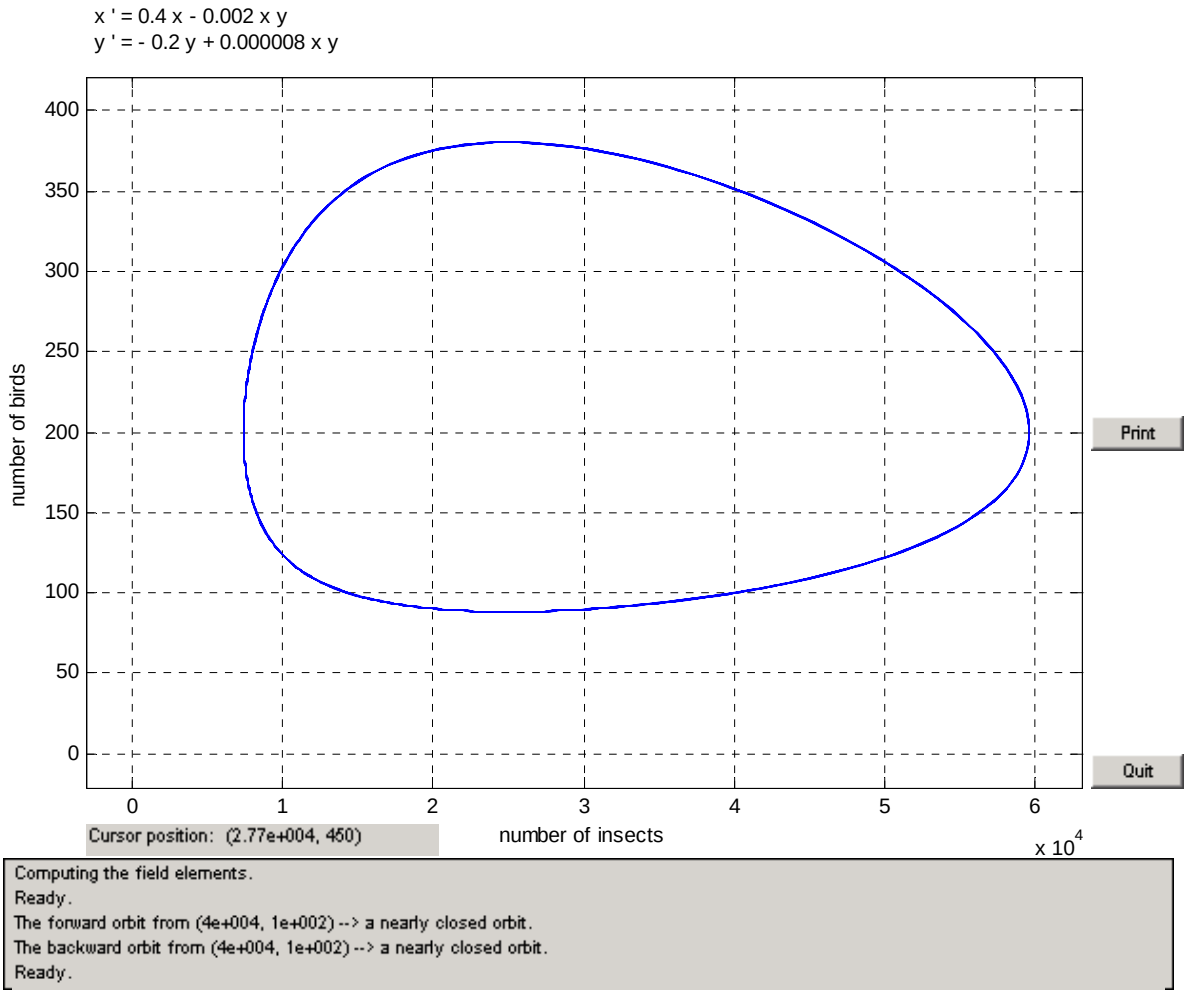
$$\frac{dy}{dx} = \frac{y(-0.2 + 0.000008x)}{x(0.4 - 0.002y)}$$

General solution in implicit form:

$$0.4\ln|y| - 0.002y = -0.2\ln|x| + 0.000008x + C \quad \textbf{or}$$

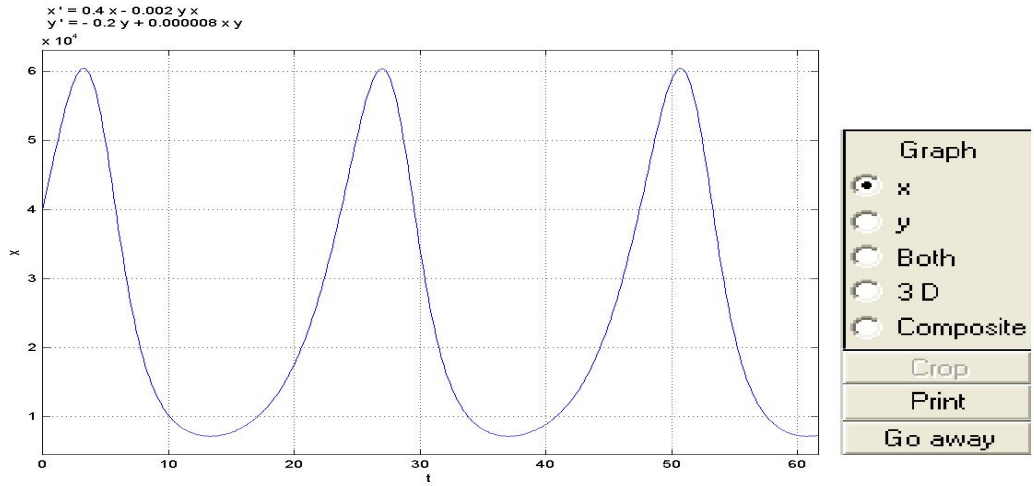
$$\frac{y^{0.4}x^{0.2}}{e^{0.002y}e^{0.000008x}} = C$$

d)

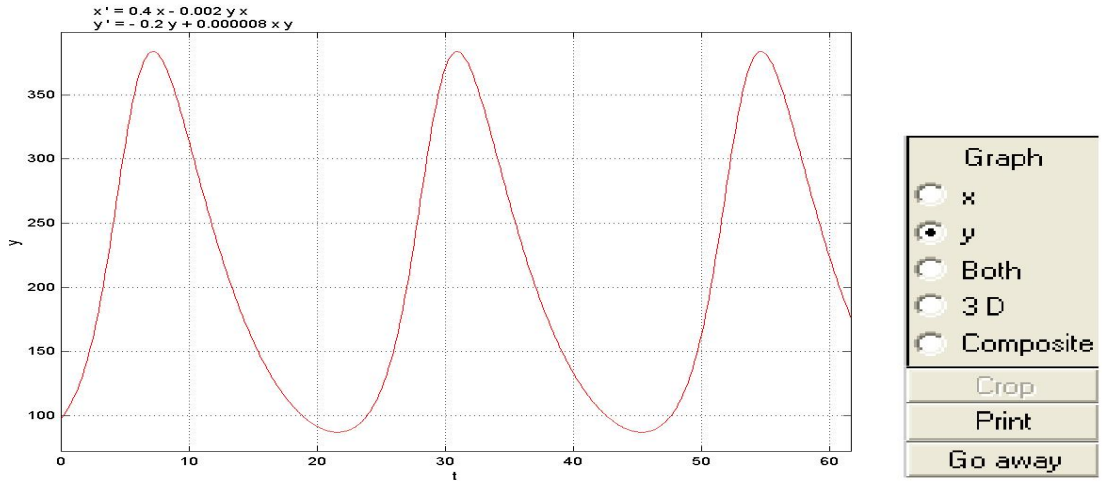


The direction on the phase trajectory is anti-clockwise. Just choose a point and substitute its coordinates into the first equation and check the sign of the derivative. E.g. x=40,000, y=100 gives x'=8,000 > 0 which means that x is increasing at that point, that is goes to the right from the point x=40,000, y=100.
Both populations change periodically with respect to time. The insect population oscillate around its equilibrium value of 25,000 from the minimum value of 7,000 to its maximum level of 60,000. The bird population oscillate around its equilibrium value of 200 from the minimum value of 80 to its maximum level of 360.

e) The number of insects versus time



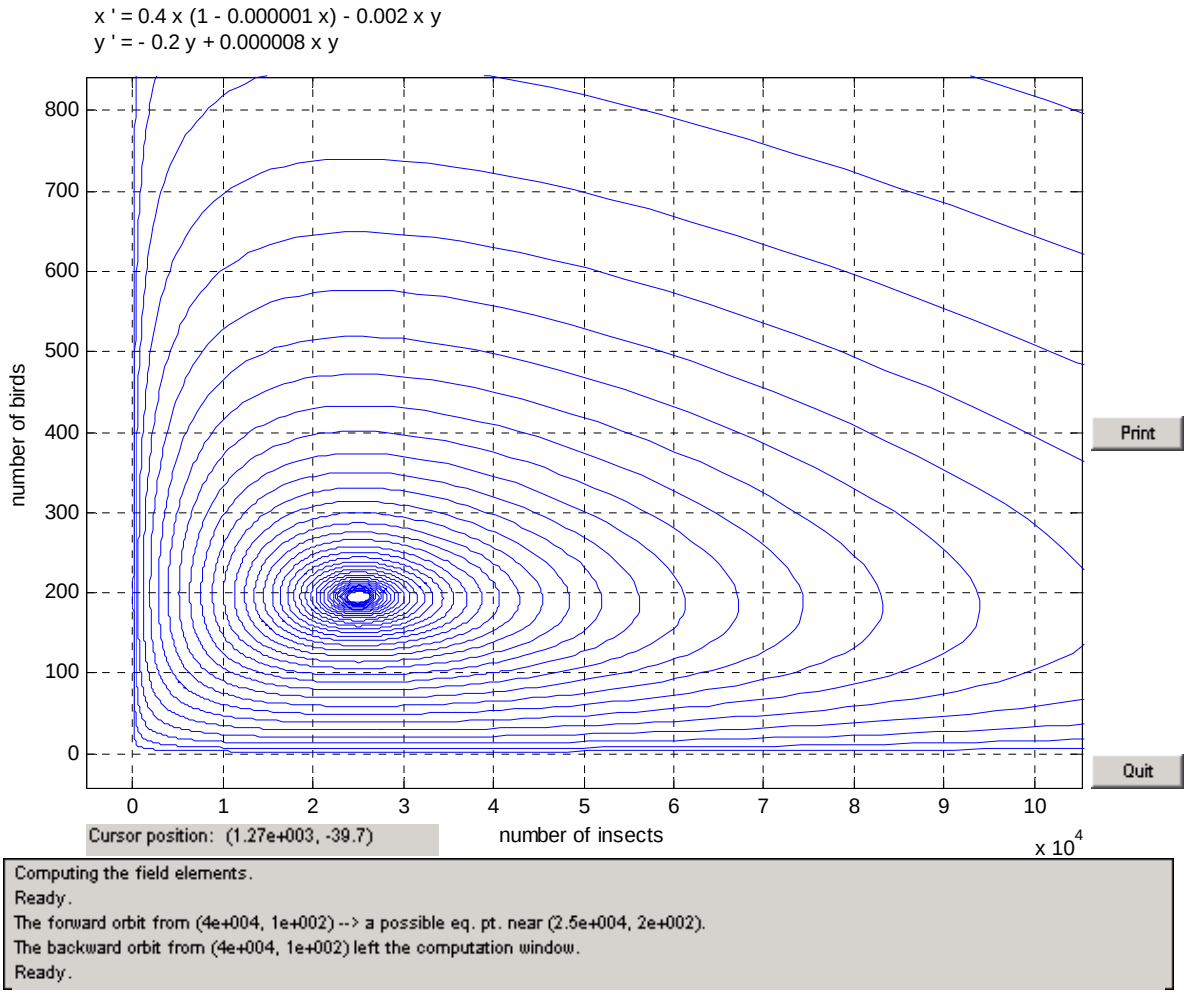
The number of birds versus time



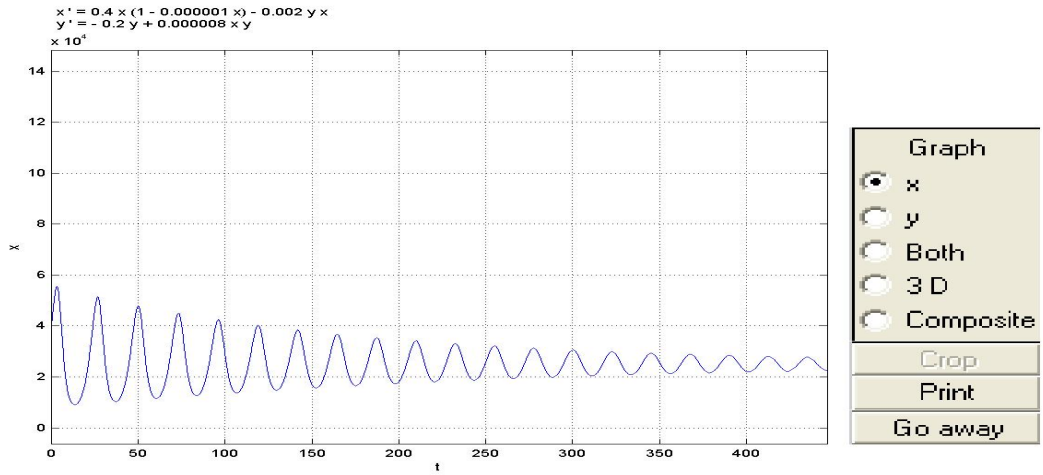
There is a horizontal shift between the two graphs. The graph of the insect population reaches its maximum values before the graph of the bird population reaches its maximum values.

f) In the absence of birds the insect population will grow according to a logistic model with the carrying capacity of 1,000,000. That means that after a long time the population of insects will be close to 1,000,000 and never exceed it.

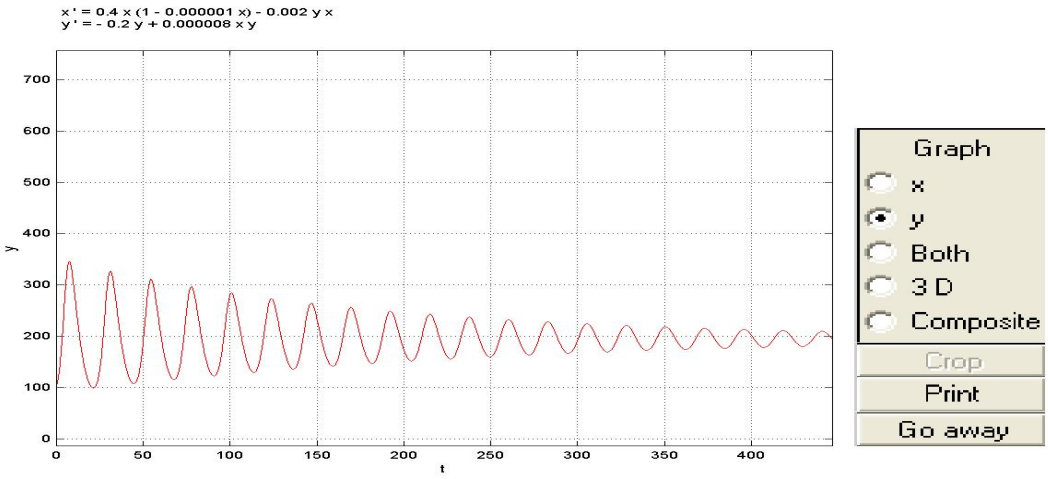
g)



The number of insects versus time



The number of birds versus time



Both populations oscillate with decreasing amplitude over time and eventually stabilize at approximately 25,000 insects and 190 birds.